

MATH 130A Review: Integration over an infinite interval

1. Determine if $\int_1^{\infty} \frac{1}{x^2} dx$ converges or diverges.

Solution:

$$\lim_{t \rightarrow \infty} \int_1^t \frac{1}{x^2} dx = \lim_{t \rightarrow \infty} \left(\frac{-1}{t} - \frac{-1}{1} \right) = 1 < +\infty$$

2. Calculate $\int_0^{\infty} x e^{-x} dx$

Solution:

$$\begin{aligned} \lim_{t \rightarrow \infty} \int_0^t x e^{-x} dx &= \lim_{t \rightarrow \infty} \left((x)(-e^{-x}) \Big|_0^t - \int_0^t (-e^{-x}) d(x) \right) \\ &= \lim_{t \rightarrow \infty} \left((t)(-e^{-t}) + \int_0^t e^{-x} dx \right) \\ &= \lim_{t \rightarrow \infty} \left(\frac{t}{-e^t} + (-e^{-t}) - (-e^{-0}) \right) \\ &= \lim_{t \rightarrow \infty} \left(\frac{t}{-e^t} + \frac{1}{-e^t} + 1 \right) \\ &= 1 \end{aligned}$$